

Let  $y = -4\sin\left(\frac{\pi}{10}x + \frac{8\pi}{5}\right) + 6$ .

SCORE: \_\_\_\_ / 13 PTS

[a] Fill in the blanks. Simplify your answers.

Middle  $y$ -value = 6

Maximum  $y$ -value = 10  
 $6+4$

Minimum  $y$ -value = 2  
 $6-4$

Amplitude = 4  
 $| -4 |$

Period = 20  
 $\frac{2\pi}{\frac{\pi}{10}} = 2\pi \cdot \frac{10}{\pi}$

Phase shift = -16  
 $\frac{\pi}{10}x + \frac{8\pi}{5} = 0 \rightarrow \frac{\pi}{10}x = -\frac{8\pi}{5} \rightarrow x = -\frac{8\pi}{5} \cdot \frac{10}{\pi}$

[b] Find the coordinates for all points corresponding to the middle, top and bottom of the graph of the function for 2 complete periods, **starting at the phase shift**.



$\frac{1}{4}$  PERIOD =  $\frac{1}{4}(20) = 5$

Point 1: ( -16 , 6 )

Point 2: ( -11 , 2 )

Point 3: ( -6 , 6 )

Point 4: ( -1 , 10 )

Point 5: ( 4 , 6 )

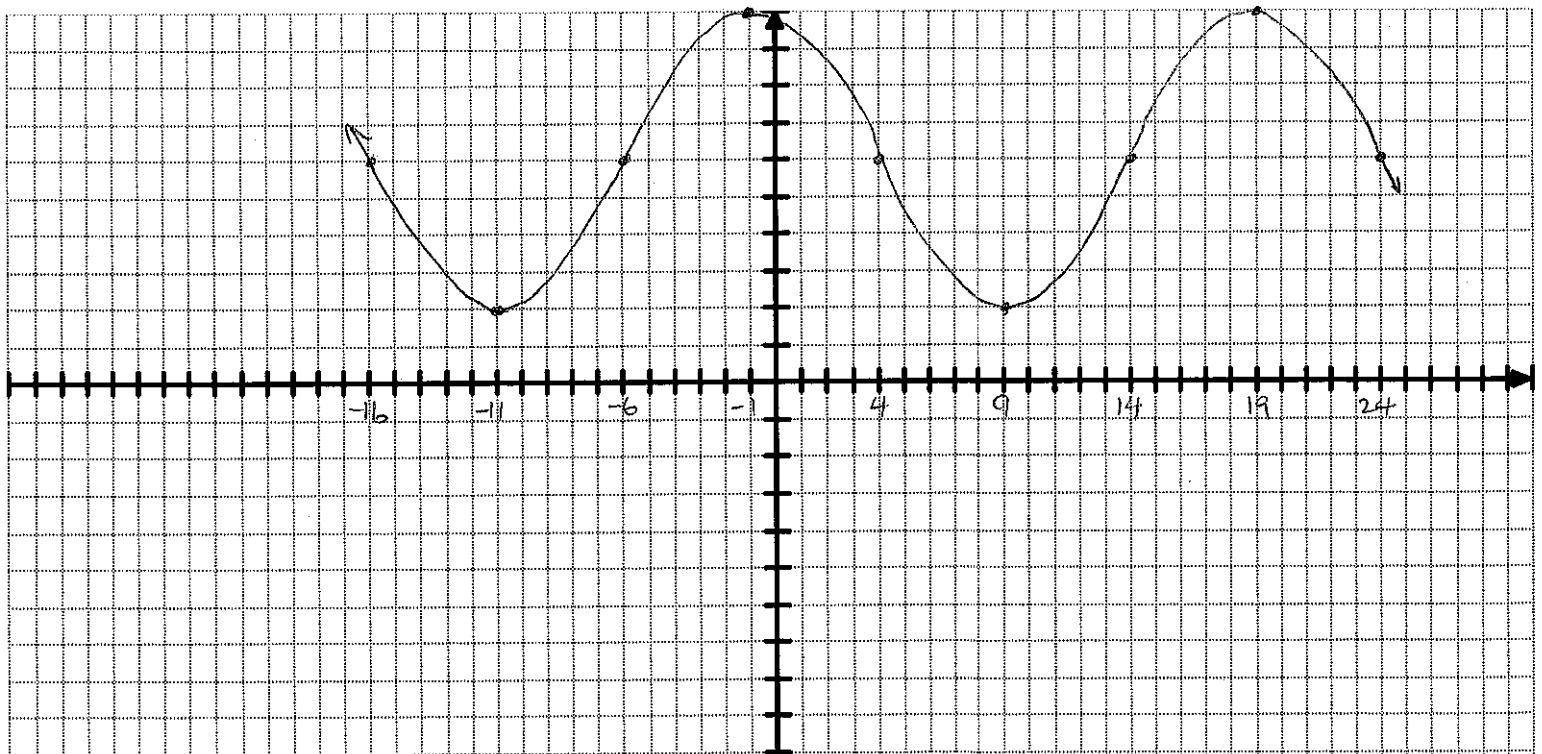
Point 6: ( 9 , 2 )

Point 7: ( 14 , 6 )

Point 8: ( 19 , 10 )

Point 9: ( 24 , 6 )

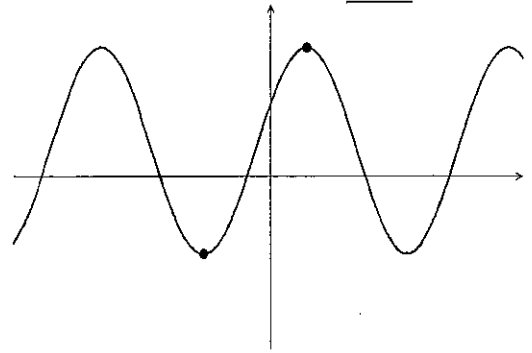
[c] On the graph paper below, sketch a detailed graph of 2 complete periods of the function using the information from [b]. You must label all  $x$ - and  $y$ -values from [b] on the appropriate axes below, and you must use a consistent scale for each axis. **You do NOT need to label each tick mark on each axis, only the ones you found in [b].**



Fill in the blanks regarding the graph on the right (**NOT DRAWN TO SCALE**). Simplify your answers.

SCORE: \_\_\_\_ / 7 PTS

NOTE: The coordinates of the two points highlighted are  $(-\frac{4\pi}{7}, -4)$  and  $(\frac{2\pi}{7}, 9)$ .



[a] Middle  $y$ -value =  $\frac{5}{2}$   
 $\frac{9+(-4)}{2}$

[b] Amplitude =  $\frac{13}{2}$   
 $\frac{9-(-5)}{2}$

[c] Phase shift =  $-\frac{4\pi}{7}$  or  $\frac{2\pi}{7}$

[d] Period =  $\frac{12\pi}{7} = \frac{2\pi}{B} \rightarrow 12\pi B = 14\pi \rightarrow B = \frac{7}{6}$   
 $\frac{1}{2}P = \frac{2\pi}{7} - (-\frac{4\pi}{7}) = \frac{6\pi}{7} \rightarrow P = \frac{6\pi}{7} \cdot 2$

[e] An equation of the graph is  $y = -\frac{13}{2} \cos \frac{7}{6}(x + \frac{4\pi}{7}) + \frac{5}{2}$  or  $\frac{13}{2} \cos \frac{7}{6}(x - \frac{2\pi}{7}) + \frac{5}{2}$   
 IF PHASE SHIFT =  $-\frac{4\pi}{7}$  IF PHASE SHIFT =  $\frac{2\pi}{7}$

These questions are about the non-sinusoidal trigonometric functions.

SCORE: \_\_\_\_ / 10 PTS

[a] Sketch 2 periods of the graphs of the following functions.

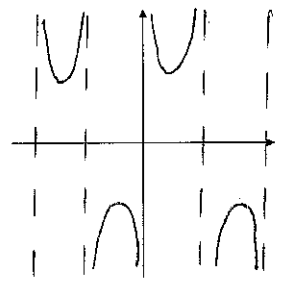
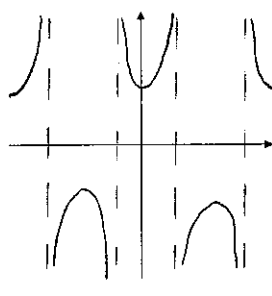
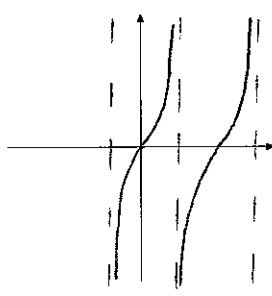
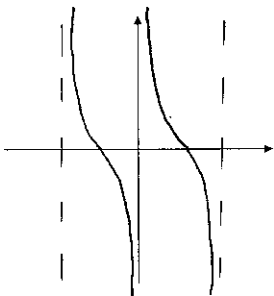
**NOTE: You only need to get the general position and shape correct. Do NOT plot points.**

$y = \cot x$

$y = \tan x$

$y = \sec x$

$y = \csc x$



[b] Fill in the blanks.

[1] As  $x \rightarrow -\pi^+$ ,  $\csc x \rightarrow -\infty$ .

[2] As  $x \rightarrow \frac{\pi}{2}^-$ ,  $\sec x \rightarrow \infty$ .

[3] The domain of  $y = \cot x$  is  $x \neq n\pi, n \in \mathbb{Z}$ .

[4] The range of  $y = \csc x$  is  $(-\infty, -1] \cup [1, \infty)$ .

[5] The equations of the vertical asymptotes of  $y = \sec x$  are  $x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$ .

[6] The period of  $y = \tan x$  is  $\pi$ .